

PERFORMANCE AND COST ANALYSIS OF LARGE CAPACITOR BANKS USING WEIBULL STATISTICS AND MTBF.*

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Summary

Inertial confinement fusion projects have required increasingly larger capacitor banks. As bank size has increased, the need for high energy density capacitors has become apparent. Higher energy densities are achieved by increasing the dielectric stress while reducing life. Acceptable capacitor life is based on system MTBF. For Nova, a 100 shot MTBF was deemed to be minimum. For a large population of capacitors, as in Nova, statistical analysis is required to evaluate system performance. The tradeoff between capacitor life and energy density can be characterized via Weibull statistics.

A method of calculating capacitor bank reliability and capacitor bank costs based on Weibull statistics is given herein.

Introduction

LLNL has been involved in laser fusion since 1971, building increasingly larger capacitor banks to power Nd:glass lasers. Shiva, built in 1977, has a 25 megajoule capacitor bank which delivers 50,000 megawatts peak power during the laser pump pulse. Nova I, presently under construction, will have a 50 megajoule capacitor bank with a peak power level of 100,000 megawatts. Nova II, scheduled for construction in 1983, will have an additional 50 megajoule capacitor bank.

The capacitors used on Shiva comprised 20% of the pulsed power system cost¹. Since then LLNL has made a concerted effort to reduce capacitor costs. The approach taken was to design a capacitor specifically for use in laser fusion, that is, low voltage reversal, moderate peak currents, moderate dc life and most importantly reduced operating life.

The way to reduce capacitor costs is to increase the energy density. Shiva, with its 25MJ occupied 8000 square feet. Nova, at 100 MJ, would have occupied 32,000 feet if the same capacitors were used. Instead LLNL in cooperation with Aerovox, General Electric, and Maxwell has developed a capacitor of approximately four times the energy density used on Shiva. These capacitors are rated 12.5 kJ, 52 μ F, at 22kV. This has reduced the capacitor cost to 12% of the total pulse power system cost.

To increase the energy density of a capacitor, the capacitor must be operated at higher dielectric stress. The higher stress reduces the lifetime, i.e., the number discharges, of the capacitor. In order to take advantage of the reduced costs of higher energy density capacitors, a realistic tradeoff between life and energy density must be established. The method used to scientifically make this tradeoff is Weibull statistics.

Background on Weibull Statistics

The question of capacitor life expectancy or probability of failure as a function of use can be described by the Weibull function.

If the capacitor failures are due to fatigue, that is wearout caused by repeated stressing of the dielectric system, a Weibull distribution function can be used to analyze the data. The form of the function is²

$$F(N) = 1 - \exp \left\{ - \left(\frac{N - N_0}{N_a - N_0} \right)^b \right\} \quad (1)$$

where F = fraction failed

N = shot life

N_0 = minimum life expectancy

N_a = characteristic life

b = Weibull slope

Equation 1 can be converted into a straight line relationship as

$$\ln \ln \frac{1}{1 - F(N)} = -b \ln(N_a - N_0) + b \ln(N - N_0) \quad (2)$$

or

$$\log \log \frac{1}{1 - F(N)} = -b \log(N_a - N_0) + b \log(N - N_0) \quad (3)$$

where

$$.3622 = \log \log(\exp)$$

Equation 3 becomes

$$\log \log \frac{1}{1 - F(N)} = a + b \log(N - N_0) \quad (4)$$

where "a" is a constant. Thus, for a given N_0 , a least-squares analysis yields an "a" and a "b" for the best straight line approximation.

The value of N_0 is chosen to minimize the sum of the squares of the deviation of the experimental points from the best straight line. From a physical standpoint $N_0 \geq 0$; for capacitor test data N_0 usually equals zero.

Given an ordered set of failures at life values of $N_1 \leq N_2 \leq \dots \leq N_k$ out of a sample size m , the data is analyzed in the following manner. For large values of $m(m > 20)$

$$F(N_i) = \frac{i}{Z}, \quad Z = m + 1, \quad i = i^{\text{th}} \text{ failure.} \quad (5)$$

If units are removed from test before they fail, and before the test is over, then the method suggested by Nyman² can be used.

$$F \left(N_{k>i} \right) = \frac{i}{Z} + \frac{k-i}{Z+1} \quad (6)$$

where

$$Z^1 = \frac{Z}{1 + \frac{j}{Z+1}}$$

j = non failed units removed from test

$\ell = m - i - j$ = units left on test.

i = number of failures before units are removed from test.

k = k^{th} failure after nonfailed units are removed from test ($k > i$)

Next, if Weibull graph paper is available, one

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plots the data points $F(N)$ vs. N on this paper. If this type of paper is unavailable, then ordinary log-log graph paper can be used by plotting

$$\log \log \frac{1}{1 - F(N)}$$

vs. N . A straight line is fitted to these data points using a least squares fit.

The slope of this straight line is the Weibull parameter "b". The characteristic life N_a can be read from the graph by observing the life corresponding to $F(N) = 1 - \exp(-1) = .632$. Having obtained this straight line, further analysis of the data may be accomplished.

As an alternative to the hand calculation method given above, a maximum likelihood estimator can also be used. Bury⁴ gives an excellent account of this method.

To develop the maximum likelihood estimators (MLE) for the parameters of the Weibull distribution, used to describe the distribution of the number of shots to failure for a capacitor, let M denote the number of units (capacitors) tested. For the j th unit, let N_j denote the number of shots the j th unit experiences during the test. One of three interpretations for N_j will be valid:

- (a) failure value: the item failed on the N_j th shot.
- (b) type I censored value: the item, yet to fail, is removed from test after N_j shots because of "runout" -- it was decided to terminate testing of the j th unit at this level of shots.
- (c) type II censored value: the item, yet to fail, is removed from test after N_j shots because another item failed on the N_j shot and it was decided to terminate testing upon this number of failures.

(The distinction between (b) and (c) is irrelevant for the analysis.) Let J denote the total number of failure values. Then $M - J$ is the number of censored values (of either type). Re-index the items such that the first J observations, N_1, N_2, \dots, N_J , represent the J failure values and the remaining observations, $N_{J+1}, N_{J+2}, \dots, N_M$ represent the $M - J$ censored values.

Assuming the lifetime (number of shots to failure) of a capacitor has a Weibull distribution, the cumulative distribution function is:

$$F(n) = 1 - \exp \left\{ - \left(\frac{n - N_0}{N_a - N_0} \right)^b \right\} \quad n \geq 0; \quad b \geq 0, \quad N_0 \geq 0, \quad N_a \geq 0 \quad (7)$$

where b - shape parameter (Weibull "slope" parameter)

N_a - scale parameter (characteristic life)
 N_0 - location parameter (Minimum life)

Assuming $N_0 = 0$, the distribution function simplifies to

$$F(n) = 1 - \exp \left\{ - \left(\frac{n}{N_a} \right)^b \right\} \quad n > 0, \quad b > 0, \quad N_a > 0 \quad (8)$$

Further, for analysis it is convenient to define

$$\lambda = \left(\frac{1}{N_a} \right)^b \quad (9)$$

Then, in terms of the parameters (λ, b) , the distribution function is

$$F(n) = 1 - \exp \left\{ - \lambda n^b \right\} \quad n > 0, \quad b > 0, \quad \lambda > 0 \quad (10)$$

Given the observations $N_1, \dots, N_J, \dots, N_M$, it can be shown that the likelihood function

$$L(N_1, \dots, N_M) \text{ is given by } L(N_1, \dots, N_M; \lambda, b) = K \exp \left\{ - \lambda \sum_{j=1}^M N_j^b \right\} b^J \prod_{j=1}^J N_j^{b-1} \quad (11)$$

where K is a complicated expression influenced by the censoring protocol, but not depending on λ and b . The maximum likelihood estimators (MLE's), $(\hat{\lambda}, \hat{b})$ of (λ, b) are obtained by maximizing $L(\cdot)$ with respect to λ and b . Using calculus techniques, we find:

1. b is the root to

$$0 = - \frac{J}{\sum_{j=1}^J N_j^b} \sum_{j=1}^M N_j^b \ln N_j + \frac{J}{b} + \sum_{j=1}^J \ln N_j \quad (12)$$

2. λ is given in terms of b as

$$= \left(\frac{1}{J} \sum_{j=1}^J N_j^b \right)^{-1} \quad (13)$$

3. Thus, the MLE, N_a , of the characteristic life N_a is

$$N_a = \left(\frac{1}{\hat{\lambda}} \right)^{1/\hat{b}} \quad (14)$$

Equation 12 is solved iteratively for \hat{b} using the data from the test program. It is convenient to use a computer for this calculation.

Regardless of the method used, to evaluate the capacitor's performance a comprehensive test program is needed.

Test Program

To qualify a capacitor build for use on Nova, a sample of 35 capacitors of each build were subjected to the following tests.

The first test was the pulse discharge test. The capacitors were discharged at the same energy, current, and reversal conditions as on Nova. Thirty of the units were tested to 20,000 shots or failure. The remaining five were tested to 100,000 shots or until 3 of 5 failed. Results of this testing were analyzed using Weibull statistics.

The pulse discharge test conditions were:

Charge voltage	24 kV
Charge time	12.5 seconds
Hold time	2.5 seconds
Discharge peak current	5 kA
Discharge voltage reversal	10%
Stabilized external temperature	25 \pm 5 degrees C

The test was operated at four discharges per minute.

The test stand used for the pulse discharge test allowed the testing of twenty individual capacitors at a time. Each capacitor had its own channel with isolation fanout, ignitron and pulse-forming network. The fanout provided diode isolation from the common power supply and also provided selectability via a "Ross" relay. The test stand was automated to facilitate testing without constant supervision.

Two of the capacitors which survived the 20,000 shot pulse discharge test were subjected to 1500 high reversal tests. The purpose of this test was to insure that the solder connections inside the capacitor were adequate.

The conditions for the reversal test were:

Charge voltage	22 kV
Charge time	22.5 seconds
Hold time	2.5 seconds
Discharge peak current	60 kA

Discharge voltage reversal	85%
Stabilized external temperature	25 \pm 5 degrees C

This test was run at a five minute repetition rate. The criteria for acceptance was that both capacitors had to survive 500 shots.

Two other capacitors which survived the 20,000 shot pulse discharge test were placed on a dc life test. These units were charged to 24 kV and held there for 1000 hrs. The criteria for acceptability was that the two units had to average 500 hrs life at voltage.

Each failed capacitor was autopsied and the failure modes identified. In addition, the capacitors were carefully examined for quality of workmanship.

Test Results

Test results for the two capacitor builds that were purchased for Nova I are summarized in Table I. Three builds from the three sources were qualified; the order for Nova I was split between the two low bidders as determined by the procedure given below.

The build from Vendor X was subjected to the 100,000 pulse discharge test twice due to an anomaly in the data from the first 35 units. As it happened, three of the four failures from the first 35 units occurred in the group of five capacitors designated to run 100,000 shots. Since this anomaly could not be accounted for, it was decided to repeat this test on five additional units and to increase the sample size to forty.

Vendor X's failure at 131 shots was not a wearout failure. Autopsy of the unit revealed the failure to be an edge track across the margin directly under the solder connection. It is thought a solder stalactite caused this premature failure. As such, this data point was not included in the Weibull analysis.

The Weibull plots for these builds are given in Figures 1 and 2. Figure one is a plot using the hand calculation method of Nyman. Figure two is a plot based on the maximum likelihood estimate. The method of Nyman gives much more pessimistic results. The MLE is much more mathematically rigorous; it has a systematic means of handling censored data. Nyman's method for censored data accounts for the removed units only in plotting the data points. The hand calculation method is easy to use, however, and gives excellent results when most of the units are tested to destruction.

MTBF and Cost Analysis

The characteristic life, N_a , and the slope, b , obtained from the Weibull plots can be used with equation 1 to characterize capacitor bank performance. Since the test data was taken at 24 kV while the bank will be running at 22 kV, the first step is to scale the data to 22 kV. This is easily accomplished by just scaling the characteristic life using the eighth power law;

$N_a^1 = (24/22)^8 N_a$. The Mean Time Between Failure is calculated by

$$MTBF = \frac{BANK\ LIFE}{BANK\ SIZE * FRACTION\ FAILED} \quad (15)$$

In order to apply equation 15 the anticipated bank life must be known. For example, it is estimated that the bank life for Nova is 5000 to 10,000 shots.

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The MTBF levels for Nova bank lives of 5000 and 10,000 shots are given in Table II. The Nova I bank will use 2000 high energy density storage capacitors. The MTBF levels are about a factor of four higher than originally expected; the design goal was a MTBF level of 100. The major reason for this increase is a smaller than expected bank resulting from 100% salvage of the Argus and Shiva capacitor banks.

The second application of Weibull analysis is the calculation of total capacitor costs. LLNL has used the following method to determine low bidder for procurement. The total cost is defined as

$$TOTAL\ COST/UNIT = INITIAL\ COST + TEST\ COST + FAILURE\ COSTS$$

$$FAILURE\ COSTS/UNIT = \frac{\#FAILED * COST\ PENALTY}{BANK\ SIZE}$$

The cost penalty is not just replacement costs but rather cost which includes the effects of down time and damage to other components resulting from possible faults, etc. For Nova, the cost penalty has been determined to be \$10,000. This number is based on repair costs, down time, and a prorated cost of amplifier repair. Amplifiers occasionally suffer damage as a result of capacitor failure. The cost penalty is a weighting factor which favors the capacitor build with the largest MTBF.

Test costs cover the infant mortality testing of capacitors purchased. LLNL will test all units to 500 shots to eliminate failures due to quality control. For Nova the test cost is estimated to be \$6.00 per unit.

Infant mortality failures drastically affect MTBF estimates. If vendors X's 131 failure point was included, the MTBF level for a bank life of 5000 shots becomes 157.

The total costs for each build is summarized in Table II. Example costs are given for both methods, Nyman and MLE. The MLE is much more optimistic in predicting MTBF. This can be attributed to the larger value of the Weibull slope that the MLE obtains. To be conservative LLNL is basing the bank costs on the more pessimistic estimates. During the course of operating Nova, data will be obtained which will determine which method gives the better estimate

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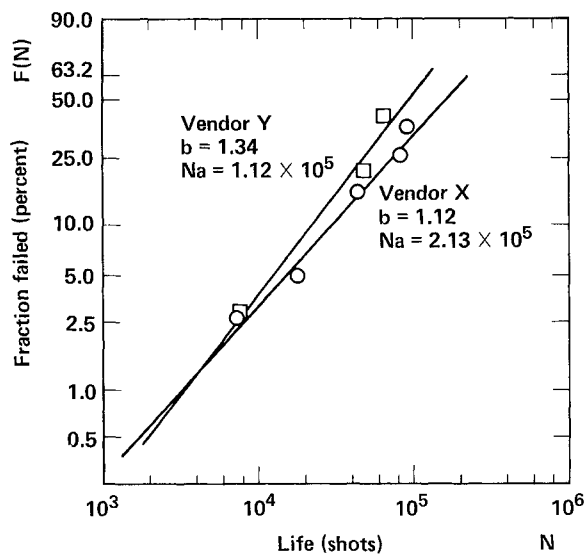


FIGURE 1 NYMAN'S METHOD

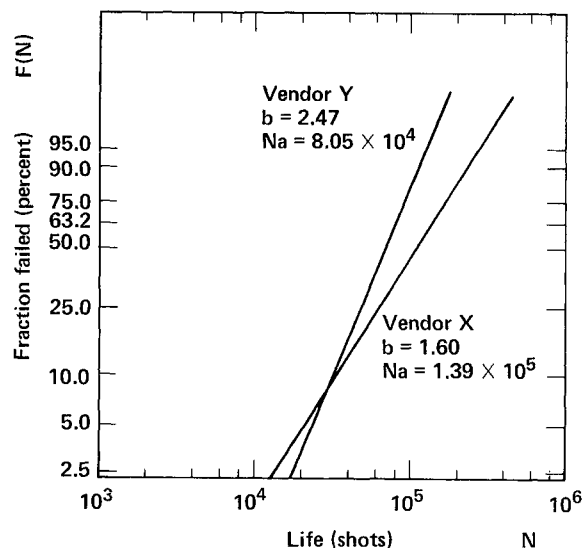


FIGURE 2 MLE METHOD

TABLE I TEST RESULTS

	VENDOR X	VENDOR Y
Total # of Cans Tested	40	35
DC Life Test	Pass, one failure T=153.4 hr. Bushing	Pass, No Failures
Ring Test	Pass - No Failures	Pass - No Failures
# of Failures-Life Test	6	3
# of Cans Removed at 20,000	29	30
# of Runout	2 @ 81,012 3 @ 101,387	2 @ 64,877
C _{ave} Measured	52.59 μ F	53.37 μ F
Failures	131 7182 17332 41930 81012 93325	7698 47775 64877

TABLE II RELIABILITY AND COST COMPARISON									
		BANK LIFE							
		5,000		10,000		5,000		10,000	
	Initial Cost	# Failure MTBF	Total Cost ₁	# Failures MTBF	Total Cost ₁	# Failures MTBF	Total Cost ₂	# Failure MTBF	Total Cost ₂
VENDOR X	650	13.95 358	\$726	30.08 332	\$806	3.18 1571	\$672	9.64 1037	\$704
VENDOR Y	666	12.12 412	\$732	30.40 329	\$824	0.37 13,354	\$674	2.07 4826	\$682

1 Nyman Method

2 MLE Method

Bank Size = 2000 Units

Cost Penalty = \$10,000 Per Failure